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E: ISSN NO.: 2455-0817 Remarking An Analisation

Some useful Results on thermal Stability Analysis of an incompressible Viscous Fluid in the Presence of Magnetic Field Confined in an Anisotropic Porous Medium

Abstract

The paper examines, within the framework of linear stability analysis with the model suggested by Brinkman, the thermal instability of an incompressible viscous fluid in the presence of magnetic field confined in an anisotropic porous medium. Uniform temperature and concentration gradients are maintained along z-axis. The interesting properties associated with magnetic field have attracted a number of different results on stability by using perturbations and normal mode analysis. In present paper, the important results obtained include different conditions of stability, existence of oscillatory modes, non-oscillatory modes, discussion for stable and unstable modes, if exist in the problem.

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Introduction

The instability of fluid flows in a porous medium under varying assumptions has been well summarized by Scheidegger[1] and Yih[2, 3]. While investigating the flows or flow instabilities through porous medium, the liquid flow has been assumed to be governed by Darcy's law[4] by most of the research workers, which neglects the inertial forces on the flow. Brinkman[5] proposed a plausible modification to Darcy's law that takes into account the viscous forces. Instability of compressible or incompressible flows has been studied extensively by a number of research workers in past few decades. In almost all such investigations, the Boussinesq's approximation is used to simplify the equations of motion.

Goel, Agrawal and Jaimala[6] examined the shear flow instability of an incompressible visco-elastic second order fluid in a porous medium in which the modified Darcy's law is replaced by the celebrated Brinkman model so that both the inertia and viscous terms are included in their usual forms. The behavior of conducting fluid is very much different in the absence and in the presence of a magnetic field. The interesting properties associated with a magnetic field, have attracted a number of research workers to work in this direction. Bansal and Agrawal[7] have studied the thermal instability of a compressible shear flow in the presence of a weak applied magnetic field. The problem of compressible shear layer in the presence of a weak applied magnetic field through porous medium has been studied by Bansal, Bansal and Agrawal[8]. The thermosolutal convection in a porous medium was studied by Nield[9], Chakrabarti and Gupta[10] and Sharma etal[11]. Khare and Sahai[12] have studied the thermosolutal convection in a heterogeneous fluid layer in a porous medium in the presence of magnetic field. Using the model as suggested by, Banerjee and Agrawal[13] investigated the thermal instability of parallel shear flows in the presence of both adverse and non-adverse temperature gradients.

In the present paper, we have examined within the framework of linear analysis, the thermosolutal instability of an incompressible viscous fluid in the presence of magnetic field confined in an anisotropic porous medium. Though some literature has been reported in which magnetic field destabilizes a wave number range known to be state in its absence. [Kent[14], Gilman[15] and Jain[16]] in most of the situations magnetic field has a stabilizing effect.Kirti Prakash and Naresh Kumar[17] examined the thermosolutal instability of a Maxwellian visco-elastic fluid in porous medium in the presence of variable gravity and suspended particles.Anshu Agarwal, Jaimala and S.C.Agrawal[18] examined the shear flow instability of visco-elastic fluid in an anisotropic porous medium.

In this paper an attempt has been made to examine the thermosolutal instability of an incompressible, viscous fluid in the presence of magnetic field and confined in a porous medium following Brinkman model.

Formulation of the problem

In this paper, the thermosolutal instability of an incompressible, viscous fluid confined in an anisotropic porous medium in the presence of magnetic field has been discussed. The fluid system has been considered between two rigid boundaries talking along x-axis and situated at $z = 0$, and $z = d$ respectively. The magnetic field has also been considered along x-axis. Uniform temperature and concentration gradients are maintained along z-axis. Equations expressing the conservation of momentum, mass, magnetic field, temperature, solute mass concentration and equation of state in Brinkman model are:

(2.1) $$\frac{\rho}{\phi}\left[\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{\phi}(v.\nabla)v\right] = -\nabla p+\rho g+\mu\left(\frac{1}{\phi}\nabla^2-\frac{1}{k_1}\right)v + \frac{1}{4\pi}(\nabla\times\mathbf{H})\times\mathbf{H},$$

(2.2) $$\nabla.\mathbf{v} = 0,$$

(2.3) $$\nabla.\mathbf{H} = 0,$$

(2.4) $$\frac{\partial \mathbf{H}}{\partial t} = \frac{1}{\phi}\nabla\times(\mathbf{v}\times\mathbf{H}),$$

(2.5) $$\frac{\partial T}{\partial t} + \frac{1}{\phi}(\mathbf{v}.\nabla)T = \kappa\nabla^2 T,$$

(2.6) $$\frac{\partial C}{\partial t}+\frac{1}{\phi}(\mathbf{v}.\nabla)C = \kappa'\nabla^2 C,$$

(2.7) $$\rho = \rho_0\left[1-\alpha(T-T_0)-\alpha'(C-C_0)\right],$$

where

$\boldsymbol{v}$ = fluid velocity,
$\boldsymbol{H}$ = magnetic field vector,
ρ = density,
μ = viscosity coefficient,
κ = thermal diffusivity,
κ' = solute diffusivity
α = thermal expansion coefficient,
α' = solute expansion coefficient,
ϕ = medium porosity,
k_1 = (k_x, k_y, k_z) medium permeability,

and g =(0, 0,–g), the gravitational acceleration. The suffix zero indicates the reference state. The basic state under investigation is, therefore, characterized by

(2.8) $$\mathbf{v} = (0,0,0), \qquad \mathbf{H} = (H_0,0,0), \qquad T = T_0 - \beta z \quad and \quad C = C_0 - \beta' z,$$

where $\beta\left(=\frac{T_1-T_2}{d}\right)$ and $\beta'\left(=\frac{C_1-C_2}{d}\right)$ may be either positive or negative. Here T_1 and C_1 (T_2 and C_2) are the temperature and concentration at the lower plate (upper plate), respectively.

Perturbations and Normal Mode Analysis

The basic state (2.8) is slightly perturbed so that every physical quantity is assumed to be the sum of a mean and fluctuating component, later designated as prime quantities and assumed to be very small in comparison to their equilibrium state values. We assume that the small disturbances are the functions of space and time variables. Hence the perturbed flow may be represented as

(3.1) $$\left.\begin{aligned} \mathbf{v} &= (0,0,0)+(u',v',w'),\\ H &= (H_0,0,0,)+(h_x',h_y',h_z'),\\ T &= T(z)+\theta',\\ C &= C(z)+\gamma',\\ \rho &= \rho_0(z)+\rho',\\ and\ p &= p_0(z)+p', \end{aligned}\right]$$

where $(u',v',w'), (h_x',h_y',h_z'), \theta', \gamma', \rho' and\ p'$ are respectively the perturbations in fluid velocity, magnetic field, temperature, concentration, density and pressure.

We substitute (3.1) into the governing equations (2.1) to (2.7) and linearize them. Analysing the disturbances into normal modes, we assume that any perturbation quantity $f'(x,y,z,t)$ is of the for

(3.2) $$f'(x,y,z,t) = f(z)\exp\left[\sigma t+i(ax+by+cz)\right],$$

where the real parts of the expressions denote the corresponding physical quantities a, b and c are the real wave numbers along the x, y and z directions respectively and σ, a time constant, is complex in general.

For the considered form of the perturbations in equation (3.2), linearized equations become

(3.3) $$\frac{\rho_0}{\phi}\sigma u = -iap-\mu\left(\frac{l^2}{\phi}+\frac{1}{k_x}\right)u,$$

(3.4) $$\frac{\rho_0}{\varphi}\sigma v = -ibp - \mu\left(\frac{l^2}{\varphi}+\frac{1}{k_y}\right)v + \frac{H_0}{4\pi}\left(iah_y - ibh_x\right),$$

(3.5) $$\frac{\rho_0}{\phi}\sigma w = -icp - g\rho - \mu\left(\frac{l^2}{\phi}+\frac{1}{k_z}\right)w + \frac{H_0}{4\pi}\left(iah_z - ich_x\right),$$

(3.6) $$au+bv+cw=0,$$

(3.7) $$ah_x+bh_y+ch_z=0,$$

(3.8) $$\sigma h_x = \frac{-H_0}{\phi}(ibv+icw),$$

(3.9) $$\sigma h_y = \frac{H_0 iav}{\phi},$$

(3.10) $$\sigma h_z = \frac{H_0 iaw}{\phi},$$

(3.11) $$\left(\sigma+\kappa l^2\right)\theta = \frac{\beta w}{\phi},$$

(3.12) $$\left(\sigma+\kappa' l^2\right)\gamma = \frac{\beta' w}{\phi}$$

(3.13) $$\rho = -\rho_0\left(\alpha\theta+\alpha'\gamma\right).$$

and

(3.14) $$l^2 = a^2+b^2+c^2 .$$

Equations (3.11) to (3.13) yield

(3.15) $$\rho = -\frac{\rho_0}{\phi}\left\{\frac{\alpha\beta\left(\sigma+\kappa' l^2\right)+\alpha'\beta'\left(\sigma+\kappa l^2\right)}{\left(\sigma+\kappa l^2\right)\left(\sigma+\kappa' l^2\right)}\right\}w.$$

After eliminating various physical quantities from these equations, we obtain the final stability equation as

(3.16) $$\frac{\rho_0}{\phi}\sigma l^2 = -\mu\left(r'm^2+rc^2\right) - \frac{a^2H_0^2 l^2}{4\pi\phi\sigma} + \frac{g\rho_0}{\phi}m^2\left\{\frac{\alpha\beta}{\sigma+\kappa l^2}+\frac{\alpha'\beta'}{\sigma+\kappa' l^2}\right\},$$

where $m^2 = a^2+b^2$,

with $r = \frac{l^2}{\phi}+\frac{1}{k}$

and $r' = r \; at \; k = k_z$

On simplifying equation (3.16), after multiplying by σ^* (complex conjugate of σ) in numerator and denominator and substituting $\sigma = \sigma_r + i\sigma_i$, *we get*

(3.17) $$\frac{\rho_0 l^2}{\phi}\left(\sigma_r+i\sigma_i\right) = -\mu\left(m^2r'+c^2r\right) - \frac{a^2H_0^2l^2}{4\pi\phi|\sigma|^2}\left(\sigma_r - i\sigma_i\right) + \frac{g\rho_0 m^2}{\phi}\left[\frac{\alpha\beta}{\left|\sigma+\kappa l^2\right|^2}\left\{\left(\sigma_r+\kappa l^2\right)-i\sigma_i\right\} + \frac{\alpha'\beta'}{\left|\sigma+\kappa' l^2\right|^2}\left\{\left(\sigma_r+\kappa' l^2\right)-i\sigma_i\right\}\right].$$

Analytical Discussion

In this section, we shall prove some important results with the help of equation (3.17).

Theorem 1: For the existence of oscillatory modes under the conditions $\beta<0$ and $\beta'<0$, show that

$$\rho_0 l^2 = \frac{a^2H_0^2l^2}{4\pi|\sigma|^2} + \frac{g\rho_0 m^2\alpha|\beta|}{\left|\sigma+\kappa l^2\right|^2} + \frac{g\rho_0 m^2\alpha'|\beta'|}{\left|\sigma+\kappa' l^2\right|^2}$$

Proof: - The imaginary part of equation (3.17) yields

(4.1)

$$\sigma_i\left[\rho_0 l^2 - \frac{a^2H_0^2l^2}{4\pi|\sigma|^2} + g\rho_0 m^2\left\{\frac{\alpha\beta}{\left|\sigma+\kappa l^2\right|^2}+\frac{\alpha'\beta'}{\left|\sigma+\kappa' l^2\right|^2}\right\}\right] = 0.$$

If β and β' are both negative then equation (4.1) can be written as

$$\sigma_i\left[\rho_0 l^2 - \frac{a^2H_0^2l^2}{4\pi|\sigma|^2} - g\rho_0 m^2\left\{\frac{\alpha|\beta|}{\left|\sigma+\kappa l^2\right|^2}+\frac{\alpha'|\beta'|}{\left|\sigma+\kappa' l^2\right|^2}\right\}\right] = 0.$$

Now, for oscillatory modes ($\sigma_i \neq 0$) we must necessarily have

$$\left[\rho_0 l^2 - \frac{a^2H_0^2l^2}{4\pi|\sigma|^2} - g\rho_0 m^2\left\{\frac{\alpha|\beta|}{\left|\sigma+\kappa l^2\right|^2}+\frac{\alpha'|\beta'|}{\left|\sigma+\kappa' l^2\right|^2}\right\}\right] = 0.$$

or $$\rho_0 l^2 = \frac{a^2H_0^2l^2}{4\pi|\sigma|^2} + \frac{g\rho_0 m^2\alpha|\beta|}{\left|\sigma+\kappa l^2\right|^2} + \frac{g\rho_0 m^2\alpha'|\beta'|}{\left|\sigma+\kappa' l^2\right|^2}$$

which proves the theorem.

Theorem 2: If the oscillatory modes ($\sigma_i \neq 0$) are unstable under the conditions $\beta < 0$ and $\beta' < 0$, then σ_r and σ_i must lie inside the circle given by

$$|\sigma|^2 = \frac{a^2 H_0^2}{4\pi\rho_0} + \frac{gm^2\alpha|\beta|}{l^2} + \frac{gm^2\alpha'|\beta'|}{l^2}$$

Proof: The imaginary part of equation (3.17) yields

$$\sigma_i\left[\rho_0 l^2 - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} + g\rho_0 m^2\left\{\frac{\alpha\beta}{\left|\sigma+\kappa l^2\right|^2} + \frac{\alpha'\beta'}{\left|\sigma+\kappa' l^2\right|^2}\right\}\right] = 0.$$

Now for oscillatory unstable modes ($\sigma_i \neq 0$, $\sigma_r > 0$), the above equation becomes

$$\rho_0 l^2 - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} - \frac{g\rho_0 m^2\alpha|\beta|}{\left|\sigma+\kappa l^2\right|^2} - \frac{g\rho_0 m^2\alpha'|\beta'|}{\left|\sigma+\kappa' l^2\right|^2} = 0,$$

or $$\rho_0 l^2 - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} - \frac{g\rho_0 m^2\alpha|\beta|}{|\sigma|^2} - \frac{g\rho_0 m^2\alpha'|\beta'|}{|\sigma|^2} < 0,$$

or $$\rho_0 l^2 < \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} + \frac{g\rho_0 m^2\alpha|\beta|}{|\sigma|^2} + \frac{g\rho_0 m^2\alpha'|\beta'|}{|\sigma|^2},$$

or $$|\sigma|^2 < \frac{a^2 H_0^2}{4\pi\rho_0} + \frac{g\rho_0 m^2\alpha|\beta|}{l^2} + \frac{g\rho_0 m^2\alpha'|\beta'|}{l^2},$$

which proves the theorem.

Theorem 3: If the oscillatory unstable modes exist under the conditions $\beta < 0$, $\beta' < 0$ and $J > 0$, then σ_r and σ_i must lie inside the circle given by

$$|\sigma|^2 = \frac{a^2 H_0^2 l^2}{4\pi J},$$

where

$$J = \rho_0 l^2 - \frac{g\rho_0 m^2}{l^4}\left[\frac{\alpha|\beta|}{\kappa^2} + \frac{\alpha'|\beta'|}{\kappa'^2}\right]$$

Proof: The imaginary part of equation (3.17) provides

$$\sigma_i\left[\rho_0 l^2 - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} + g\rho_0 m^2\left\{\frac{\alpha\beta}{\left|\sigma+\kappa l^2\right|^2} + \frac{\alpha'\beta'}{\left|\sigma+\kappa' l^2\right|^2}\right\}\right] = 0.$$

Now, for oscillatory modes ($\sigma_i \neq 0$), under the conditions $\beta < 0$ and $\beta' < 0$, the above equation becomes

$$\rho_0 l^2 - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} - g\rho_0 m^2\left[\frac{\alpha|\beta|}{\left|\sigma+\kappa l^2\right|^2} + \frac{\alpha'|\beta'|}{\left|\sigma+\kappa' l^2\right|^2}\right] = 0,$$

or

$$\rho_0 l^2 - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} - g\rho_0 m^2\left[\frac{\alpha|\beta|}{\kappa^2 l^4} + \frac{\alpha'|\beta'|}{\kappa'^2 l^4}\right] < 0,$$

or $$J - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} < 0,$$

where

$$J = \rho_0 l^2 - \frac{g\rho_0 m^2}{l^4}\left[\frac{\alpha|\beta|}{\kappa^2} + \frac{\alpha'|\beta'|}{\kappa'^2}\right]$$

or $$|\sigma|^2 < \frac{a^2 H_0^2 l^2}{4\pi J},$$

or $$|\sigma|^2 = \frac{a^2 H_0^2 l^2}{4\pi J},$$

which proves the theorem.

Theorem 4:

If the oscillatory modes exist, then show that

$$\left(\sigma_r + \frac{\kappa l^2}{1-S}\right)^2 + \sigma_i^2 < \kappa^2 l^4 \frac{S}{(1-S)^2},$$

where $$S = 4\pi g\rho_0 \frac{m^2\alpha\beta}{a^2 H_0^2 l^2},$$

Proof: The imaginary part of equation (3.17) yields

$$\sigma_i\left[\rho_0 l^2 - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} + g\rho_0 m^2\left\{\frac{\alpha\beta}{\left|\sigma+\kappa l^2\right|^2} + \frac{\alpha'\beta'}{\left|\sigma+\kappa' l^2\right|^2}\right\}\right] = 0.$$

For oscillatory modes, the above equation reduces to (4.2)

$$\rho_0 l^2 - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} + g\rho_0 m^2\left[\frac{\alpha\beta}{\left|\sigma+\kappa l^2\right|^2} + \frac{\alpha'\beta'}{\left|\sigma+\kappa' l^2\right|^2}\right] = 0,$$

For the validity of the above equation, we must necessarily have

$$\frac{g\rho_0 m^2\alpha\beta}{\left|\sigma+\kappa l^2\right|^2} - \frac{a^2 H_0^2 l^2}{4\pi|\sigma|^2} < 0,$$

or $$|\sigma|^2 S - \left|\sigma_r + \kappa l^2\right|^2 < 0,$$

where $$S = 4\pi g\rho_0 \frac{m^2\alpha\beta}{a^2 H_0^2 l^2},$$

or

$$\sigma_r^2 S + \sigma_i^2 S - \sigma_r^2 - \kappa^2 l^4 - 2\sigma_r \kappa l^2 - \sigma_i^2 < 0,$$

or

$$\sigma_r^2 \left(S-1\right) + \sigma_i^2 \left(S-1\right) - \kappa^2 l^4 - 2\kappa l^2 \sigma_r < 0,$$

or

$$\sigma_r^2 + \sigma_i^2 - \frac{2\kappa l^2}{S-1}\sigma_r - \frac{\kappa^2 l^4}{S-1} < 0,$$

or

$$\left(\sigma_r + \frac{\kappa l^2}{1-S}\right)^2 + \sigma_i^2 < \kappa^2 l^4 \frac{S}{\left(1-S\right)^2},$$

where $S = 4\pi g \rho_0 \dfrac{m^2 \alpha \beta}{a^2 H_0^2 l^2}$,

which proves the theorem.

Remark: The another condition for the validity of the equation (4.2) we can show that in similar way

$$\left(\sigma_r + \frac{\kappa' l^2}{1-S}\right)^2 + \sigma_i^2 < \kappa'^2 l^4 \frac{S}{\left(1-S\right)^2},$$

where $S = 4\pi g \rho_0 \dfrac{m^2 \alpha' \beta'}{a^2 H_0^2 l^2}$,

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